HIGHER SYMMETRIES IN ABSTRACT STABLE HOMOTOPY THEORIES

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The theory of derivators goes back to Alex Heller, Alexander Grothendieck, Jens Franke, and others, and it addresses the following slogan.

"The passage from an abelian category to its derived category results in a loss of information."

A closely related, very prominent problem is the non-functoriality of the cone construction. Similarly, traces of endomorphisms in derived categories fail to be additive, and this was Grothendieck's original motivation to introduce derivators as an enhancement – the main idea being that one should keep track of derived categories of diagram categories in a systematic way. Derivators axiomatize this idea, thereby providing a minimal extension of the classical derived categories to a categorical framework encoding the rich and non-trivial calculus of derived (co)limits.

Having in mind the obvious question "Why would people at ICRA potentially care?", I allow myself to offer three different answers.

- First, stable derivators yield a representation theoretic explanation of the axioms of a triangulated category – thereby allowing for an export of representation theoretic thinking to other areas of pure mathematics.
- Second, stable derivators allow us to extend certain derived equivalences of quivers over fields to more general contexts like representations over rings, over schemes, over dgas, or over ring spectra. Examples of such higher symmetries are abstract reflection equivalences at sources or sinks in quivers. (j/w Jan Šťovíček)
- Third, conjecturally, abstract representation theory might become useful in calculations. A promising playing ground for such higher symmetries is provided by abstract cubical homotopy theory – a study of cubical shapes in stable derivators. (j/w Falk Beckert)

The goal of these lectures is to shed more light on these three answers, one at a time.

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